§26. Collisional Effects on Coherent Structures of Zonal Flows and Turbulent Transport

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In this work, the collisional effects on nonlinear coherent structures of zonal flows in a toroidal plasma are investigated. They weaken the flows and enhance turbulent transport. Expressions for the zonal flow velocity, scale length and the turbulent transport coefficients are obtained. In the limit of small collisional flow damping, the turbulent transport coefficient remains finite. A phase diagram is obtained, which shows the parameter domain where the zonal flow has finite amplitude.

The growth of zonal flows in the presence of drift-wave turbulence is discussed, taking into account the drive of zonal flows by turbulence and the nonlinear suppression mechanism for the flow. One of the damping mechanisms was the wavenumber dispersion of the drift waves, in conjunction with the zonal flow shearing. An additional damping mechanism associated with the secondary toroidal flow was also included. The viscous breaking of this secondary flow is an essential damping mechanism of the zonal flows. Taking into account these processes, and employing a perturbation expansion of the wave kinetic equation to third order, we obtain a nonlinear equation as

$$\frac{\partial}{\partial t} U + D_{rr}\left(\frac{\partial^2}{\partial r^2} U + K_0^{-2}\frac{\partial^4}{\partial r^4} U\right)$$
$$- D_3 \frac{\partial^2}{\partial r^2} U^3 - \mu_{\|}\left(1 + 2q^2\right)\frac{\partial^2}{\partial r^2} U - \nu_t U = 0$$

where U is the vorticity of the zonal flow, the zonal flow growth is denoted by

$$D_{rr} = \frac{c^2}{B^2}\int d^2k \frac{R(K,\Omega) k_\theta^2 k_r}{\left(1 + k_\perp^2\rho_s^2\right)^2}\frac{\partial N_k}{\partial k_r}$$

and $R(K,\Omega) = i/\left(\Omega - K\,\partial\omega/\partial k_r + i\Delta\omega_k\right)$ is the response kernel [1] and ν_t is a rate of damping by the ion-ion collisions. (The zonal flow has slow variation in space and time as $\exp(iKr - i\Omega t)$, and $\Delta\omega_k$ is the nonlinear broadening.) For a wide spectrum of turbulent fluctuations, one has $R(K,\Omega) \to 1/\Delta\omega_k$. In such a case, $R(K,\Omega)$ is approximately even with respect to k_r, and

$$D_3 = -\frac{c^2}{B^2}\int d^2k \frac{R(K,\Omega)^3 k_\theta^4 k_r}{\left(1 + k_\perp^2\rho_s^2\right)^2}\frac{\partial^3 N_k}{\partial k_r^3}$$

denotes the magnitude of nonlinear stabilization effect. Here, $\mu_{\|}$ is the turbulent shear viscosity for the flow along the field line and q is the safety factor. Both the zonal flow driving coefficient D_{rr} and the shear viscosity $\mu_{\|}$ are determined by the drift wave spectrum. The ratio between them, i.e., $\mu \equiv \mu_{\|}\left(1 + 2q^2\right)D_{rr}^{-1}$ is a function of the spectral shape of drift wave turbulence and geometrical factors such as q.
The fluctuation level of the drift-wave turbulence is determined on the fast time scale, and is expressed by use of the gradient of the zonal flow dV/dr. We write the effect of the zonal flow vorticity on the transport coefficients as

$$D \equiv \frac{D_{rr}}{D_{rr}(0)} = \frac{1}{1 + W u^2}.$$

where $W = \frac{1}{2}\tau_{ac}^2 v_z^2 K_0^2 (1-\mu)^2$, and $D_{rr}(0)$ is the value of D_{rr} in the absence of the zonal flow. The shear viscosity $\mu_{\|}$ has the same dependence, $\mu_{\|} = D\,\mu_{\|}(0)$. The coefficient $\hat{\nu} = \nu D^{-1}$ denotes the role of collisional damping.

The nonlinear equation for the zonal flow is solved and the flow velocity is obtained as a function of other control parameters [2]. For instance, we find that the zonal flow is suppressed in the case of frequent collisions,

$$\nu_t D_{rr}(0)^{-1} K_0^{-2}(1-\mu)^{-2} > 1/4$$

A phase diagram is shown in Fig.1

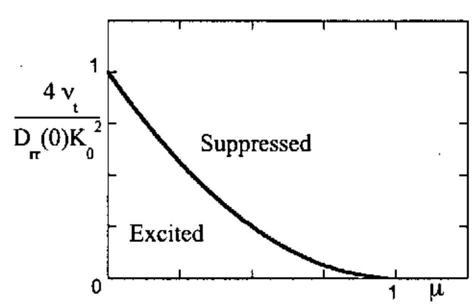

Fig.1 Parameter domain where the zonal flow is excited.

References
[1] A. Smolyakov and P. H. Diamond: Phys. Rev. Lett. **84** (2000) 491
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